

Midterm 1 Conceptual Review

Math 791, Spring 2019

- Midterm 1 is in class on **Tuesday, March 5**.
 - The exam will cover the parts of §0.3, §§7.1 – 7.4, §7.5, and §§9.1 – 9.3 that we have studied, along with the *Quotient Rings worksheet*. (Please consult me, or see the Daily Update for a brief rundown of our topics).
 - The best preparation is to **practice, practice, practice** working and re-working problems. This includes **quiz problems** and **book problems**.
 - To create your own **practice exam**, choose 1-2 problems from each section that cover a variety of the topics listed.
 - Check out **extended office hours** that will be posted on the course website.
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Basics on rings (§7.1)

- **Concepts:** Ring, ring axioms, commutative ring, additive/multiplicative identities, negative/additive inverse, subring, (multiplicative) inverse, familiar rings ($\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$), zero divisor, unit, (integral) domain, field, nilpotent element, direct product of rings
 - **Goals:** Determine whether a set with given operations is a ring, determine whether a given ring has certain properties, determine the zero divisors and units of a ring, prove statements about rings using the ring axioms, determine whether a given subset of a ring is a subring
 - **Homework problems:** Prove *Proposition 1 (3)-(4)*; §7.1: #1 - 3, 5, 7, 14, 15, 17, 18, 26
 - **Additional problems:** §7.1: #4, 6, 9, 21, 30
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Examples of rings (§§7.1 - 7.2, , §0.3)

- **Concepts:** Rings of functions, the ring $\mathbb{Z}/n\mathbb{Z}$, matrix rings, polynomial rings, degree of a polynomial
 - **Goals:** Perform calculations in different rings, determine the zero divisors and units of a given ring, prove statements about certain types of rings
 - **Homework problems:** §7.1: #6, 13; §7.2: #1, 2, 3, 4, 8
 - **Additional problems:** §7.2: #5 - 7
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Ring homomorphisms (§7.3)

- **Concepts:** Ring homomorphism, kernel, image, ring isomorphism, natural projection, evaluation homomorphism
 - **Goals:** Verify whether a given function between rings is a ring homomorphism, study/prove properties of ring homomorphisms, classify all ring homomorphisms between given rings
 - **Homework problems:** §7.3: #1, 2, 4 - 7, 13, 17, 24, 26; *Daily Update problems* #1 - 4
 - **Additional problems:** §7.3: #11, 12, 16, 32
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Ideals (§§7.3 - 7.4)

- **Concepts:** Left/right ideal, ideal, sum of ideals, product of ideals, power of an ideal, principal ideal, finitely-generated ideal, ideal generated by a set, prime ideal, maximal ideal, kernel of a ring homomorphism is an ideal, ideals of the integers
 - **Goals:** Determine whether a subset of a ring is an ideal; compute the sum, product, or powers of ideal(s); describe and use principal, finitely-generated ideals, and ideals generated by a subset of a ring; identify prime and maximal ideals in a ring; prove statements about or involving ideals
 - **Homework problems:** §7.3: #8 - 10, 17, 20, 29, 34, 36; §7.4: #7 - 9, 12, 13
 - **Additional problems:** §7.3: #18, 19, 22; §7.4: #4, 10, 11
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Quotient rings (§7.3 and *Quotient Rings worksheet*)

- **Concepts:** Quotient ring modulo an ideal, coset, ring homomorphism from a quotient ring, natural projection homomorphism onto a quotient ring
 - **Goals:** Understand and describe a certain quotient ring, construct or verify a ring homomorphism from a quotient ring, determine properties of quotient rings, prove statements about quotient rings
 - **Homework problems:** *Quotient Rings worksheet*; §7.4: #14, 16
 - **Additional problems:** §7.4: #17, 25, 31
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Isomorphism Theorems for Rings (§7.3)

- **Concepts:** *First, Second, Third, and Fourth/Lattice Isomorphism Theorems for Rings*
 - **Goals:** Understand, prove, and apply the Isomorphism Theorems for Rings
 - **Homework problems:** *Prove Second Isomorphism Theorem, Daily Update problems #2 - 4*
 - **Additional problems:** *Prove Third Isomorphism Theorem*
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Chinese Remainder Theorem (§7.6)

- **Concepts:** Comaximal ideals, *Chinese Remainder Theorem*
 - **Goals:** Apply the *Chinese Remainder Theorem*, verify whether two ideals are comaximal
 - **Homework problems:** §7.6: #1 - 5, 7
 - **Additional problems:** §7.6: #6
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Polynomial rings (§9.1 - 9.3)

- **Concepts:** Zero divisors and units in a polynomial ring, ideals of polynomial rings, polynomial ring in several variables, *Division Algorithm* for polynomials over a field, reducible and irreducible polynomials, *Unique Factorization* for polynomials over a field, *Root Theorem* for polynomials over a field
- **Goals:** Carry out calculations in polynomial rings, prove statements about the polynomial ring $R[x]$ based on properties of R , apply properties of polynomial rings over fields, determine whether or not a polynomial is irreducible
- **Homework problems:** §9.1: #5; §9.2: #1, 5, 6; §9.3: #2
- **Additional problems:** §9.2: #2, 7