Midterm 1 Conceptual Review

Math 500: Intermediate Analysis, Spring 2017

- The exam will focus on **Chapters 1 and 2**, and **Section 3.1**, in our text.
- You may use both sides of a 3" x 5" **note card** (or paper) during the exam (but **no calculator**).
- You are **highly encouraged** to come to the instructor with questions! Look for **office hours** posted on the course website. If you can't make them, feel free to **make an appointment**.

- **Sets** (§1.1)
  - **Concepts**: set, element of a set, subset, empty set, interval, union of sets, intersection of sets, complement of sets, product of sets
  - **Goals**: Use and interpret set notation properly; prove statements about sets, including equality or inclusion; calculate intersections, unions, and complements of sets
  - **Homework problems**: §1.1: #2, 4, 5
  - **Additional practice problems**: §1.1: #1, 3, 6
- **Functions between sets** (§1.1)
  - **Concepts**: function between sets, domain of a set, image of (a set under) a function, preimage/inverse image of a set under a function, onto function, one-to-one function
  - **Goals**: Prove statements about functions between sets, including about the image or preimage/inverse image of sets under a function; determine the image or preimage/inverse image of set under a function; determine whether a function is one-to-one or onto
  - **Homework problems**: §1.1: #9, 13
  - **Additional practice problems**: §1.1: #8, 11, 14
- **Induction** (§1.2)
  - **Concepts**: The Principle of Mathematical Induction, base case, inductive hypothesis
  - **Goals**: Prove statements for all natural numbers (or all natural numbers greater than a fixed integer) using the Principle of Mathematical Induction, including statements about sequences defined recursively
  - **Homework problems**: §1.2: #8, 9, 14
  - **Additional practice problems**: §1.2: #10, 11, 13
- **Axiomatic definitions** (§§1.2 - 1.3)
  - **Concepts**: Peano's axioms for the natural numbers, the axioms of a commutative ring (satisfied by the integers), the additional axiom needed to be a field (that the rational numbers satisfy), the axioms of an ordered field (that the rational numbers satisfy)
  - **Goals**: Prove statements about the above object by only using the axioms defining them (Note that if you are required to use these axioms on an exam, you would be given them)
  - **Homework problems**: §1.2: #2; §1.3: #8-10
  - **Additional practice problems**: §1.3: #3, 7
- **The real numbers** (§1.4)
  - **Concepts**: Dedekind cut, upper bound, the completeness axiom, the Archimedean property
  - **Goals**: Prove statements about Dedekind cuts from the definition, find an upper bound / the set of all upper bounds for a non-empty set of real numbers
  - **Homework problems**: §1.4: #1 - 3, 10
  - **Additional practice problems**: §1.4: #4, 12

- **Bounds, infima, and suprema** (§1.5)
  - **Concepts**: least upper bound, greatest lower bound, supremum, infimum (possibly of functions)
  - **Goals**: Identify whether a set of real numbers is bounded above/below; identify the least upper bound/greatest lower bound of a set if it exists, and prove this is the case; identify the supremum/infimum of a set of real numbers and prove this is the case; prove statements about bounds and suprema/infima
  - **Homework problems**: §1.5: #2, 7, 9
  - **Additional practice problems**: §1.5: #1, 5, 8, 11
- **Limits of sequences** (§§2.1 - 2.4)
  - **Concepts**: triangle inequality, sequence, limit of a sequence (finite/infinite), convergence
  - **Goals**: Identify whether the limit of a sequence exists and find it if it does, prove that the limit of a sequence is a number/does not exist/is infinite, prove statements about limits, use theorems about limits to make conclusions
  - **Homework problems**: §2.1: #1 - 5, 8, 11; §2.2: #2, 5, 6; §2.3: #4, 5,
  - **Additional practice problems**: §2.1: #6, 7, 9, 10; §2.2: #1, 3, 4, 7; §2.3: #1, 3, 6
- **Monotonic sequences** (§2.4)
  - **Concepts**: non-increasing sequence, non-decreasing sequence, monotonic/monotone sequence, Monotonic Convergence Theorem
  - **Goals**: identify whether a sequence is monotonic, prove that a sequence is monotonic (possibly by induction), show that a sequence is bounded, apply the Monotonic Convergence Theorem to conclude that a sequence converges
  - **Homework problems**: §2.4: #1 - 3, 5, 8, 9, 11
  - **Additional practice problems**: §2.4: #10, 12
- **Subsequences and Cauchy sequences** (§2.5)
  - **Concepts**: subsequence, Bolzano-Weierstrass theorem, Cauchy sequence
  - **Goals**: identify convergent subsequences and their limits, apply the Bolzano-Weierstrass theorem, prove that a sequence is Cauchy, prove that a sequence converges by showing it is Cauchy (and vice versa)
  - **Homework problems**: §2.5: #5 - 9, 11
  - **Additional practice problems**: §2.5: #10, 12
- **Limit superior and limit inferior** (§2.6)
  - **Concepts**: monotonic sequences $\{i_n\}$ and $\{s_n\}$ associated to a sequence, limit supremum/limit infimum of a sequence, subsequential limit
  - **Goals**: calculate the limit supremum and limit infimum of a sequence, with proof; prove statements about limit suprema and limit infima
  - **Homework problems**: §2.6: #1, 3, 6
  - **Additional practice problems**: §2.6: #2, 5, 8, 12
- **Continuity** (§3.1)
  - **Concepts**: natural domain of a function, continuity of a function at a specific point of its domain, continuity of a function on its entire domain, characterization of continuity in terms of sequences
  - **Goals**: identify the natural domain of a function (or composition of functions); decide whether a given function is continuous at a point/for all point in its domain, and prove it
  - **Homework problems**: §3.1: #1, 3, 4, 8, 9
  - **Additional practice problems**: §3.1: #5, 11