Final Review Session Problems

Math 290: Elementary Linear Algebra

Fall 2018

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1. Given the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{bmatrix}$,
   (a) Find a basis for the row space.
   (b) Find a basis for the null space.
   (c) Verify the rank-nullity theorem for A.

2. Given the function
$$T : P_1(\mathbb{R}) \to P_2(\mathbb{R})$$
   given by $T(ax + b) = ax^2 + bx + (a + b)$,
   (a) Verify that T is a linear transformation.
   (b) Find a basis for the range of T.
   (c) Find a basis for the kernel of T.
   (d) Verify the rank-nullity theorem for T.

3. Given the set
$$W = \{(2s - t, s, t) \mid s, t \in \mathbb{R}\},$$
   (a) Show that W is a subspace of $\mathbb{R}^3$.
   (b) Find a basis for W.
   (c) Determine the dimension of W.

4. Which of the following are bases for $\mathbb{R}^3$?
   (a) $S = \{(7, 0, 3), (8, -4, 1)\}$
   (b) $S = \{(2, 1, -2), (-2, -1, 2), (4, 3, -4)\}$
   (c) $S = \{(0, 0, 0), (1, 0, 0), (0, 1, 0)\}$
   (d) $S = \{(1, 1, 1), (0, 1, 1), (1, 0, 0)\}$

5. Which of the following are linear transformations?
   (a) $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ given by $T(A) = \det(A)$.
   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x, 1)$.
   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(\mathbf{v}) = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{v}$.

6. Which of the following are isomorphic vector spaces?
   (a) $W = \{\text{diagonal } 2 \times 2 \text{ matrices}\} \subseteq M_{2\times 2}(\mathbb{R})$
   (b) $\mathbb{R}^3$
   (c) $P_2(\mathbb{R})$