# Midterm 2 Conceptual Review

## MATH 290, Fall 2018

- Midterm 2 is in class on Thursday, November 8.
- The exam is **cumulative**, but will focus on  $\S2.5$ ,  $\S3.1 3.4$ , and  $\S4.1 4.4$ .
- We will have time on Tuesday's class to review some practice problems.
- The best preparation is to **practice**, **practice**, **practice** problems. This includes **quiz problems**, **WebAssign problems**, and **book problems**.
- **Solutions** to odd-numbered additional practice problems can be found in the back of the textbook, and solutions to WebAssign problems can be found through WebAssign.
- You may use a **calculator** and  $3'' \times 5''$  **notecard**, but cannot replace any class methods.
- To create your own **practice exam**, choose 1-2 problems from each section that cover a variety of the topics listed.
- Check out **extended office hours** that will be posted on the course website.

#### $\S2.5$ : Markov chains

- **Concepts**: Stochastic matrix, matrix of transition probabilities, state matrix, Markov chain, steady state of a Markov chain, regular stochastic matrix, regular Markov chain, absorbing state, absorbing Markov chain
- **Goals**: Determine state matrices associated to a Markov chain, find the steady state of a Markov chain (if it exists), determine whether a stochastic matrix is regular, determine whether a Markov chain is absorbing
- **Practice problems**: §2.5: #7, 9, 13, 23, 25, 45; Ch 2 Review: #51, 63, 71

#### $\S3.1$ : The determinant of a matrix

- Concepts: Determinant of a matrix, minors of a matrix, cofactors of a matrix
- **Goals**: Find the determinant of a  $2 \times 2$  matrix, find the minors and cofactors of a matrix, use expansion by cofactors to find a determinant, find the determinant of a triangular matrix
- Practice problems: §3.1: #15, 18, 29, 41, 43, 67; Ch 3 Review: #17, 47

#### $\S3.2$ : Determinants of elementary operations

- **Concepts**: Effect of row and column operations on determinants, conditions that yield zero determinant
- **Goals**: Use elementary row or column operations to evaluate a determinant, recognize conditions that yield zero determinants
- Practice problems: §3.2: #15, 31, 39, 42, 45; Ch 3 Review: #21

#### $\S3.3$ : Properties of determinants

- **Concepts**: Formulas for determinants of matrices obtained from other matrices, the notion that a matrix is nonsingular if and only if its determinant is nonzero
- **Goals**: Find the determinant of a matrix product and a scalar multiple of a matrix, find the determinant of an inverse matrix and recognize equivalent conditions for a nonsingular matrix, find the determinant of the transpose of a matrix
- Practice problems: §3.3: #5, 13, 17, 21, 27, 33, 39, 47, 67; Ch 3 Review: #43, 76

#### $\S3.4$ : Applications of determinants

- **Concepts**: Matrix of cofactors, adjoint of a matrix, Cramer's rule
- **Goals**: Find the adjoint of a matrix and use it to find the inverse of the matrix; use Cramer's Rule to solve a system of n linear equations in n variables; use determinants to find area, volume, and the equations of lines and planes
- Practice problems: §3.4: #7, 17, 27, 29, 35, 37, 41, 51, 59, 61; Ch 3 Review: #35, 75, 77

### §4.1: Vectors in $\mathbb{R}^n$

- **Concepts**: Vectors in the plane  $\mathbb{R}^2$ , in 3-space  $\mathbb{R}^3$ , and in *n*-space  $\mathbb{R}^n$ ; components of a vector; vector addition; zero vector; scalar multiplication of a vector
- **Goals**: Represent a vector as a directed line segment; perform basic vector operations in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ , and represent them graphically; perform basic vector operations in  $\mathbb{R}^n$ , write a vector as a linear combination of other vectors
- **Practice problems**: §4.1: #1, 17, 24, 31, 57

## $\S4.2$ : Vector spaces

- **Concepts**: Vector space, vector space operations, zero element in a vector space, the vector spaces  $\mathbb{R}^n$ , vector spaces of matrices, vector spaces of functions, properties of scalar multiplication on vector spaces
- **Goals**: Define a vector space, recognize some important vector spaces, determine whether or not a given set with given operations is a vector space
- Practice problems: §4.2: #2, 9, 11, 47, 24, 25, 29, 31, 37, 41, 49, 50; Ch 4 Review: #25, 26

#### $\S4.3$ : Vector subspaces

- Concepts: Subspace of a vector space, test for being a subspace
- **Goals**: Determine whether a given subset of a vector space V is a vector subspace of V, determine vector subspaces of  $\mathbb{R}^n$
- Practice problems: §4.3: #3, 7, 12, 16, 18, 19, 24, 27, 29, 35, 39, 43, 44; Ch 4 Review: #22, 23

## $\S4.4$ : Spanning sets and linear independence

- **Concepts**: Linear combination of vectors in a vector space, spanning set of a vector space, the span of a set in a vector space, linear dependent/independence
- **Goals**: Write a linear combination of a set of vectors in a vector space, determine whether a set of vectors in a vector space is a spanning set, find the span of a set in a vector space, determine whether a set of vectors in a vector space is linearly independent or linearly dependent, write a vector as a linear combination of other vectors
- **Practice problems**: §4.4: #7, 17, 27, 37, 45, 51, 57, 61, 63, 64