# Final Exam Conceptual Review

# Math 145, Fall 2017

- The Final Exam is on Wednesday, December 13 from 4:30 7:00 pm in 301 Snow.
- The exam will be **cumulative**, and review sheets from material before Midterm 2 are posted online.
- The post Midterm 2 material consists of  $\S$  5.3 5.8, 7.1, and mathematical induction (which will only appear on the final exam as extra credit).
- You may use both sides of a 8.5" x 11" sheet of paper during the exam.
- You may use a calculator, but its use should **never** replace a method from class.
- You are **highly encouraged** to come to the instructor with questions! Look for **office hours** posted on the course website. If you can't attend them, feel free to **make an appointment**.

## Antiderivatives $(\S5.3)$

- *Concepts*: Antiderivative / indefinite integral, power rule for antiderivatives, linearity of antiderivatives, basic rules for antiderivatives, differential equation (with initial conditions)
- *Goals*: Find antiderivatives using basic antiderivative rules, solve differential equations (with initial conditions)
- Homework problems: §5.3: #3, 9, 22, 31, 41, 42
- Additional practice problems: §5.3: #17, 35, 40, 46, 53, 63

## Fundamental theorem of calculus, Part I (§5.4)

- Concepts: Fundamental theorem of calculus, Part I (FTC I); antiderivative of  $y = \frac{1}{x}$
- Goals: Understand and apply FTC I to calculate definite integrals, understand and apply the antiderivative of  $y = \frac{1}{r}$
- Homework problems: §5.4: #9, 19, 33, 41, 47, 55, 62
- Additional practice problems: §5.4: #29, 34, 56, 64

### Fundamental theorem of calculus, Part II $(\S5.5)$

- Concepts: Fundamental theorem of calculus, Part II (FTC II); area functions  $A(x) = \int_a^x f(t) dt$
- *Goals*: Understand and apply FTC II, interpret area functions and find derivatives of their composition with other functions
- Homework problems: §5.5: #1, 5, 9, 11, 13, 19, 23, 25, 26, 31, 33, 37, 39, 44, 45, 48
- Additional practice problems: §5.5: #3, 13, 26, 32, 41, 43

#### Net change as integral of rate of change $(\S5.6)$

- Concepts: Net change as integral of rate of change, total displacement/distance traveled
- Goals: Calculate/estimate net change as integral of rate of change, calculate total displacement/distance traveled as integral of velocity/absolute value of velocity

- *Homework problems*: §5.6: #1, 7, 11, 19
- Additional practice problems: §5.6: #5, 9, 15, 17

### Substitution method for integrals $(\S5.7)$

- Concepts: Substitution method for integrals (reverse rule of chain rule for derivatives)
- Goals: Apply substitution method to compute antiderivatives and definite integrals
- Homework problems: §5.7: #7, 11, 15, 23, 27, 45, 73, 77, 89, 91, 94, 97, 98
- Additional practice problems: §5.7: #12, 17, 22, 41, 67, 80, 91, 101

#### Integration by parts $(\S7.1)$

- Concepts: Integration by parts method for integrals (reverse rule of product rule for derivatives)
- *Goals*: Apply integration by parts, combined with other methods, to compute antiderivatives and definite integrals
- Homework problems: §7.1: #3, 7, 11, 13, 21, 25, 29, 35, 51, 46, 55, 77
- Additional practice problems: §7.1: #27, 38, 52, 60, 63, 75, 76

#### Mathematical induction (extra credit) (Childs book)

- Concepts: The principle of mathematical induction used to prove a statement about all natural numbers (or all  $n \ge a$  for some integer a)
- Goals: Apply the principle of mathematical induction to prove a statement
- Homework problems: §7.1: #1, 2, 3, 8, 13, 14 in Childs
- Additional practice problems:
  - 1. Prove that for all integers n > 1,

$$1 + 3 + 3^2 + \dots + 3^{n-1} = 3^n - 1.$$

- 2. Prove that for all positive integers  $n, 3^{4n} 1$  is a multiple of 5.
- 3. Prove that  $2^{2n} > n^4$  for all integers  $n \ge 4$ .
- 4. Prove that for all integers  $n \ge 1$ ,

$$1^{4} + 2^{4} + 3^{4} + \dots + n^{4} = \frac{n(n+1)(2n+1)}{(3n^{2} + 3n - 1)} 30.$$